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# Micrometeors and the Paul trap

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### **Abstract**

In the late 1950's, a Paul trap was investigated as a way to electrically charge micron size iron particles, for a laboratory micrometeor simulator. Particles could be charged but not high enough. The alternating quadrupole electric field was found to form stable arrays of charged particles, inside the trap, which could be resonantly excited. Positive and negative ions were also trapped and detected by their absorption of radio frequency power at their frequency of oscillation in the three dimensional effective potential well. These studies were done under a then active federally mandated aerospace independent research and development (IR&D) program; which, along with published papers, produced a (now public domain) patent which includes particle arrays, particle resonance schemes, particle damping, and particle expulsion. (Int J Mass Spectrom 190/191 (1999) 1–7) © 1999 Elsevier Science B.V.

*Keywords:* Meteor simulation; Mathieu equations; Paul trap; Charged particle arrays; Resonances

## **1. Introduction**

Meteor impacts on space vehicles were a concern of the fledging American Space Program of the 1950's. It was known that 10–20 tons of meteoric dust impact the earth each day at speeds from 7 km/s (earth escape) to 70 km/s [1]. Most meteors are micron in size with kinetic energies of only 0.25–25  $\mu$ J, but power fluxes of  $10^{10}$ – $10^{12}$  W/cm<sup>2</sup>. Accordingly, an experimental program was launched to study micrometeor phenomena in the laboratory. Light gas guns existed which accelerated millimeter sized particles to barely 7 km/s. Electrostatics offered the most promising way of achieving true meteor speeds, provided a method could be found to electrically charge dust to close to its limiting positive charge. For iron,  $E_{\text{surface}} \le 200 \text{ MV/cm}$ , which for a 2  $\mu$ m diameter particle means a charge to mass ratio of 100 C/kg, and a velocity of 15 km/s after acceleration through 2 MV. The micrometeor accelerator project was started at the Ramo Wooldrige Corporation (RW), Los Angeles, California, which also managed the development of the first American Ballistic Missile (ABM) defense system.

The micrometeor project was directed by David B. Langmuir. Others at the start included Herbert C. Corben. Robert V. Langmuir (California Institute of Technology), Haywood Shelton, and myself.

Robert Langmuir knew magnetic strong focusing techniques from nuclear accelerator work. He was also familiar with the then new linear alternating electric quadrupole work of W. Paul in Germany [2–4]. Robert Langmuir conceptualized bending the rods of a linear quadrupole into a circle (forming a "racetrack"), then shrinking the structure until the

Dedicated to J.F.J. Todd and R.E. March in recognition of their original contributions to quadrupole ion trap mass spectrometry.

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inner rod disappeared. The new configuration was similar to a linear quadrupole, except for a two to one ratio in the strengths of the orthogonal electric fields (in the *z* and *r* cylindrical directions), while still giving a force on a charged particle proportional to its distance from the origin. The electrode structure consisted of hyperbolic end caps and a hyperbolic ring electrode; the now classical Paul trap, with the following Cartesian interior space potential;

$$
V(x, y, z, t) = \frac{V_{\text{ac}}}{Z_o^2} \left[ z^2 - \left( \frac{x^2 + y^2}{2} \right) \right] \sin \Omega t \quad (1)
$$

which results when an alternating voltage source of  $2V_{\text{ac}}$  (peak to peak) of angular frequency  $\Omega$  (radians/s) is connected between the ring electrode and the two (electrically connected) end caps, separated from one another by a distance of  $2Z_o$ . If time is frozen, when the end caps are at  $+V_{ac}$  (with the ring electrode at  $-V<sub>ac</sub>$ ), a positively charged particle of mass *m* would vibrate back and forth through the origin harmonically at an angular frequency of

$$
\omega_{\text{peak}-z} = \omega_{p-z} = \sqrt{\frac{2 \text{eV}_{\text{ac}}}{m Z_o^2}}
$$
 (2)

where *e* is the charge on the particle. When the voltage reverses the particle accelerates unstably toward an end cap at an exponential rate. When the voltage varies sinusoidally with time, the particle spends more time going toward the origin than away, with the result that it oscillates at a much lower frequency than the drive frequency. This new frequency is identified in McLachlan's [5] classic book on Mathieu equations, as a subharmonic of the drive or as the " $\beta$ " frequency. It is usually written as " $\omega_{\beta}$ ." Analysis shows that when  $\Omega > \omega_{p-z}$ ,

$$
\omega_{\beta-z} = \frac{1}{\sqrt{2}} \left[ 2 \left( \frac{e}{m} \right) \frac{V_{\text{ac}}}{Z_o^2} \right] \left[ \frac{1}{\Omega} \right] = \frac{1}{\sqrt{2}} \left( \frac{\omega_{p-z}^2}{\Omega} \right) \quad (3)
$$

for the *z* direction. Superimposed on this frequency is a small ripple at  $\Omega$  which is 180 $^{\circ}$  out of phase with the drive. The particle's motion can be shown to be

$$
Z(t) \cong A_o[\cos(\omega_\beta t + \delta)] - Z(t)(\omega_p/\Omega)^2 \sin \Omega t]
$$
\n(4)

in spite of the fact that it is being focused and defocused at the frequency  $\Omega$ . This is the bases of "strong focusing," which can be demonstrated with an "upside down" pendulum [6]; such as a physical (rod) pendulum attached to the blade holder of an electrical hand held oscillating (jig) saw.\*

A similar analysis, for either the *x* or *y* directions of Eq. (1) (the "*r*" polar coordinate), shows that the particle vibrates at half the frequency in the *z* direction. This 2:1 ratio in sub harmonic frequencies means that a trapped particle should execute in space a "figure 8" Lissajous pattern about the origin, with the drive frequency superimposed as a ripple.

The Paul structure was proposed as a means of holding onto a dust particle while it was being electrically charged, prior to acceleration by a high voltage source.

## **2. Initial dust trapping experiments**

A hyperbolic electrode structure of characteristic dimension  $Z<sub>o</sub> = 0.6425$  cm = 0.25 inch was machined out of aluminum. Holes were drilled in the "ring" electrode to view the chamber's interior and to introduce electrons. Holes were also drilled in the centers of the cap electrodes, for the introduction of dust. The interior hyperbolic surfaces were painted with Aquadag (black electrical conducting colloidal carbon). This structure was setup in a vacuum bell jar with connections to an external high voltage transformer, controlled by a variable transformer at either 60 or 400 Hz.

An electron gun (the exposed filament of a flashlight bulb) was mounted, in front of one of the holes in the ring electrode. A piece of copper (window) screen was set up above the top cap electrode. An arm (operated through a vacuum feed through) was fitted with a camel's hair paint brush. With the whole system under vacuum ( $\approx 10^{-5}$  Torr), the brush was

<sup>\*[6]</sup> analyses the "upside down" pendulum in detail. The authors similarly conclude that the driven pendulum is confined in an effective potential well. References include A. Stephenson (1908), P. Kapitza, H.C. Corben (1960) and L. Landau, E. Lifschitz (1960).

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dipped into a pot of carbonyl iron powder (spherical micron sized pure iron particles) and "painted" along the screen. Particles were released which fell under gravity through the hole in the cap electrode into the interior of the trap, where they were charged by the electron beam. Looking through the view port one saw luminous tracks which settled down in time, showing the expected "figure 8" Lissajous pattern. In many cases more than one particle was captured, which also settled down in several minutes into beautiful three dimensional arrays. When the drive voltage was lowered, the trapped particles separated. When the drive voltage was increased, the particle array compressed until suddenly it appeared to "melt," the particles still trapped, but swarming about on wild orbits inside the quadrupole [7]. Dropping the drive voltage reestablished the ordered array.

Application of a repulsive dc voltage to the ring electrode could be used to squeeze the particles out through the holes in the cap electrodes, until only one particle was left, which could then be used for beam charging experiments. The electron source was also replaced by a small homemade ion source, operating with either hydrogen or argon gas. We found that particles could be charged up to the Mathieu  $\beta = 1$ limit ( $\omega_{\beta-z} = \Omega/2$ ), at which point they became unstable and were driven out of the trap by the beginning of the first Mathieu parametric resonance. Charging a particle also increased its orbit making it difficult to track and control. High *e/m* values required higher  $\Omega$ 's.

A new ac drive system was next assembled from a wide band (20–20 kHz) audio oscillator, a 100 W audio tube amplifier, and audio matching transformers (operated "backwards" by connecting the low impedance windings to the amplifier's low impedance speaker output). This arrangement generated  $V_{ac} \leq 1$ kV from 20 Hz to 18 kHz. We also changed from iron to aluminum particles which reflected light better and were easier to view. A carbon arc light source was procured to illuminate the trapped particles. In addition, a new electronic dust injector was positioned under the quadrupole structure. The improved system was moved to a smaller metal vacuum chamber with a window over the top cap electrode, for axial views of the interior. With the carbon arc and brighter particles, the trapping phenomena could even be photographed with ASA 400 speed films. Both still and movies were recorded of much of the phenomena [7].

Physicists are taught Earnshaw's theorem (every dc potential well is accompanied by an orthogonal saddle), which was clearly being circumvented by the ac particle trap. Successful charged particle containment resulted in some lively discussions on the longterm stability of a trapped dust particle. It was thought that collisions with the background gas would cause a particle to slowly drift in phase relative to the drive voltage, resulting in its eventually being hurled out of the trap. It was thought that a dust particle would eventually loose charge (through gas collisions) and simply fall out of the trap with time. To test these criticisms, a particle was trapped and monitored for three months. It neither got lost nor lost a measurable fraction of its initial 0.005 C/kg charge.

The phenomena of multicharged particle trapping inspired H.C. Corben to give some memorable discourses on J.J. Thomson's "raisin bun" theory of the atom [8].

Charging particles to high *e/m* values in the trap proved difficult. Charge to mass ratios of one C/kg were achieved, only 1% of what was required for a successful meteor accelerator. A even simpler method was discovered by H. Shelton; namely, a miniature antenna with a 10–50 micron tip (made by etching tungsten wire to a needle, as in a field emission microscope system, followed by sparking the tip with a charged capacitor, to form a microscopic ball on the end) [9]. The antenna was biased in vacuum to high voltage while iron particles dropped on the tip. Particle charge to mass ratios of 10 C/kg (surface field strength of 25 MV/cm) were achieved with electrodes having end tip radii of 25 microns at 20 kV. Velocities of 3 km/s were achieved with a 150 kV source [9], which was eventually replaced by a 2 MV Van de Graaf system that achieved earth escape velocities [10]. Later, the 2 MV system became the injector for a 10 MV audio frequency linear meteor accelerator, which achieved true meteor speeds [11]. This final system was used over the greater part of decade for

micrometeor impact and luminosity studies. The RW meteor accelerator has since been dismantled, having fulfilled its mission. Today the impact effects of micrometeors are more easily simulated by focusing the  $\approx 0.5$  J-10 ns pulse from a *O*-switched solid state laser onto a surface with a lens.

## **3. Mandated aerospace independent research development (IR&D)**

Success of the contact micro particle charging scheme could have ended further electrodynamic charged particle trapping studies, had not RW been required (at that time) to expend  $\approx$  1–2% of its gross federal contract funds on "fundamental research." As a result, investigations on the quadrupole system were continued. My own interest derived from earlier graduate work that involved trapping electrons in a magnetic field with electrically biased end caps.

The evacuated audio dust trapping apparatus was simple. It was, in a sense, an analogue computer for studying the behavior of single and multicharged particles in an alternating quadrupole field. I was able to explore the stability boundaries of the Mathieu *a*-*q* diagram. Direct current voltages could be added to the end caps to cancel the effect of gravity on the trapped particles. Additional alternating voltages could be applied for the purposes of resonating particles at their  $\beta$ <sub>z</sub> or sub harmonic frequency in the *z* direction, for the purpose of measuring the particle *e/m*'s, and for tracing out constant  $\beta$ <sub>z</sub> lines in the Mathieu  $a_z - q_z$ plane. Beat phenomena were observed, when the resonating frequency was slightly less or more than the particle's  $\beta$ <sub>z</sub> sub harmonic frequency. The resonance experiments gave us confidence that with no  $V_{\text{dc}}$ , the particles were trapped in an effective "real" three dimensional potential well of the form;

$$
V_{\text{well}} = \frac{q_z V_{\text{ac}}}{4Z_o^2} \left( z^2 + \frac{r^2}{4} \right) \tag{5}
$$

where

$$
q_z = 4\left(\frac{e}{m}\right)\left(\frac{V_{\text{ac}}V}{Z_o^2}\right)\frac{1}{\Omega^2} \tag{6}
$$

is the dimensionless Mathieu quantity in the  $a_{\tau}$ - $q_{\tau}$  plane. The maximum equipotential surface is the ellipse

$$
Z_o^2/2 = z^2 + r^2/4 \tag{7}
$$

that just touches the surface of the ring electrode, with a electrical potential well depth of

$$
V_{\text{max well}} = q_z V_{\text{ac}}/8 \tag{8}
$$

Well depths of 50 V effective potential occur when  $q_z = 0.5$  and  $V_{ac} = 1000$  V peak. Observations of damped arrays of many charged particles show a 1/4 relationship between the *z* and *r* directions of the trapped cloud.

Gauss' law can be used to calculate the maximum amount of charge,  $Q_{\text{max}}$ , which can be contained by a Paul trap

$$
Q_{\text{max}} \cong \sqrt{2} \{ \pi \epsilon_o q_z V_{ac} Z_o \} \tag{9}
$$

where  $\epsilon_o = 8.87 \times 10^{-10}$  Farads/m, with  $Q_{\text{max}}$  having the units of  $C/m<sup>3</sup>$ . Division of this quantity by the volume of the equipotential surface of maximum value  $(\sqrt{2}[4\pi/3]Z_o^3)$  gives the charge density. For ions, division by the electronic charge *e* gives the maximum atomic particle density that a Paul trap can handle:

$$
N_{\text{max}} = 3\epsilon_o q_z V_{\text{ac}} / 4eZ_o^2 \tag{10}
$$

For singly charged ions with  $q_z = 0.5$ ,  $Z_o = 1$  cm, and  $V_{\text{ac}} = 1000 \text{ V}, N_{\text{max}} = 2 \times 10^{10} \text{ ions/cm}^3$ . However, if  $V_{ac} = 100$  kV and  $Z_o$  is only 1 mm, then  $N_{\text{max}} = 2 \times 10^{14}$  ions/cm<sup>3</sup>, a value of interest to some recent controlled plasma fusion schemes.

We also found that particles could be resonated by applying the resonating voltage in series with the drive. This can also be shown (from McLachlan) to be a parametric resonance (exponential growth with time) instead of the linear resonance (linear orbit growth with time) across the end caps. All of these observations were included in one of the first papers on the Paul trap [12]. This paper included photographs of individual particle orbits (including one that shows the 2:1 Lissajous pattern) as well as photographs of damped arrays of many particles.

A cubical electrode structure, driven by three phase voltages, was described in a companion paper, in the same issue [13]. The  $3\phi$  structure gives a more uniform three dimensional ac potential well.

A recently found paper, by H. Strabel [14], describes the entrapment of charged water droplets at atmospheric pressure by a single ring electrode driven at 1–5 kV at 50 Hz. This paper is strictly observational with no explanation of the phenomena. It does show that trapping of charged particles by ac voltages is not particularly dependent on electrode shape, as is known today. The quadrupole potential of Eq. (1), however, provides a sound analytical bases for the phenomena. We too knew that dust particles could be trapped at atmospheric pressures. However, one cannot go from atmospheric pressures to low vacuum without passing through the Paschen minimum, which causes the electrodes to arc. Damping is discussed by McLachlan [5], as well as experimentally by Whetten [15], and again more recently by Hendricks and Wuerker [16].

Two of our particle trapping photographs [12] were included in W. Paul's 1989 Nobel prize lecture [17], which also included pictures of stable arrays of trapped ions, detected with resonant laser light [18]. Ray March's contributions were also cited [19].

## *3.1. Atomic particle experiments*

Paul's [20,21] early trapping and detection of ions was reproduced at RW. For our experiments, amateur radio equipment was used to generate the  $V_{ac}$  at radio frequencies, while a "marginal oscillator" (common to Nuclear Magnetic Resonance experiments, NMR) was used to sense the ions inside the Paul trap. A radio amateur transceiver and a linear amplifier gave 300 W of matched power at 2, 4, 7, 14, 28, or 32 MHz. This combination drove an inductively coupled resonant tank circuit that was connected to the trap's ring electrode, through a vacuum feed through. Radio frequency voltages as high as 5 kV could be generated.

Trapped ions were detected by their effect on a second (much lower frequency) tuned circuit connected across the end caps; namely, a Pound Knight (PK) marginal oscillator [22]. In our case the absorption of radio frequency power by the trapped ions was within the condenser (formed by the trap's end caps and an external tuning condenser) rather than in the



Knight marginal oscillator (at  $\omega/2\pi = 2.50$  MHz) connected across the end caps of a Paul trap. For this example;  $Z_0 = 0.624$ cm,  $\Omega/2\pi = 14.243$  MHz, and  $2V_{\text{ac}} = 1075$  V<sub>peak</sub> (upper picture) and  $2V_{ac} = 1245$  V<sub>peak</sub> (lower picture). The horizontal axis was a 55 V wide—30 Hz modulation in the Mathieu  $a<sub>z</sub>$  value.

coil of the PK detector, as in conventional NMR. The end caps were shorted together at the drive  $(\Omega)$ frequency by series resonant "traps," to keep the drive from blanking out the PK detector. The output of the PK detector was connected to the vertical amplifier of a cathode ray oscilloscope (CRT) while the series modulator  $(V_{dc})$  was connected to the horizontal amplifier. Fig. 1 shows two typical traces, due to the absorption of rf power from the detector at  $\omega_z = 2.50$ MHz by H<sup>+</sup> ions in the trap ( $Z_0 = 0.624$  cm = 0.25 in), when  $\Omega/2\pi = 14.243$  MHz and  $2V_{ac} = 1075$  or 1245 V. The background hydrogen gas pressure was  $2 \times 10^{-5}$  Torr with 80  $\mu$ A of 150 V electron current directed along the axis of the trap. The horizontal sweep was a 30 Hz sine wave of 50 V width, which modulated the Mathieu  $a_z$  value cyclically. The reso-



 $V_{AC}$  = 650  $V_{peak}$ 

 $V_{AC}$  = 508  $V_{peak}$ 

 $V_{AC}$  = 452  $V_{peak}$ 

Fig 2. Photographs showing the simultaneous detection of  $He<sup>+</sup>$  and He<sup>-</sup> ions in a Paul trap by a marginal oscillator tuned to  $\omega/2\pi$  = 0.536 MHz,  $\Omega/2\pi = 6.99$  MHz, background He pressure 2  $\times$  $10^{-5}$  Torr, 50  $\mu$ A electron beam (150 V) and horizontal sweep of 45 V peak to peak at 30 Hz and for different  $V_{\text{ac}}$  drive voltages: 452  $V_{peak}$  (lowest picture, He<sup>+</sup> resonance left of the He<sup>-</sup> resonance), 461  $V_{\text{peak}}$ , 508  $V_{\text{peak}}$  (middle picture), 580  $V_{\text{peak}}$ , and 650  $V_{\text{peak}}$ , (top picture with  $He<sup>+</sup>$  resonance now on the right).

nant curves had a measured width of  $\Omega/\Delta\Omega = M/\Delta M$ of only 38. The resonant frequency was also a factor of  $3/2$  (approximately  $\sqrt{2}$ ) lower than the single particle value because of space charge effects. We

also detected  $H_2^+$ ,  $H_3^+$ ,  $He^+$  along with  $He^-$ . Fig. 2 shows a set of resonance's when helium was being admitted. The two resonance's  $(He<sup>+</sup>$  and  $He<sup>-</sup>)$  moved toward one another as  $V_{ac}$  increased, even disappearing when superimposed.

We even procured a war surplus (APT-5) aircraft "radar jammer" which was converted into a CW 50 Watt 300 MHz source for trapping electrons. A quadrupole was mounted in the center of a slab resonator and driven with the "jammer," producing a plasma within the electrode structure.

We were attempting to make the containment research "useful," because the quadrupoles were seen then as somewhat of "a solution looking for a problem to solve." One idea was a new low noise microwave amplifier, similar to Adler's cyclotron parametric amplifier [23] with Cuccia [24] input and output couplers. At the time, we had the idea of replacing the heavy magnetic field solenoid with a microwave linear ac quadrupole electric field. The concept never got beyond the proposal stage.

A Paul trap with a single charged macro particle was even considered as a new inertial guidance system, using the inertial displacement of a trapped particle because of acceleration of the whole system.

Mass spectroscopic applications of the Paul trap never received much interest at RW, because of the poor resolution of the marginal oscillator method of ion detection.\*

## *3.2 Departures*

Concurrently (1959) RW was being criticized for unfairly giving contracts to its subsidiaries. To reduce such conflicts of interest, the Air Force reorganized the company. The meteor project was moved to the other side of Los Angles County. I joined the remaining group (then called Space Technology Laboratories or STL), only to rejoin D.B. Langmuir's group when

<sup>\*</sup>Our understandings on the Paul trap did become the basis of U.S. patent application, which included mass spectroscopy, end cap resonance, parametric resonance, damping of the trapped particles with a background gas, three phase structures, etc. [25]. The patent was filed August 27, 1959 and was granted November 27, 1962, with 20 claims. It became public domain in 1978.

it returned as a part of the (now) TRW Corporation. My active involvement with quadrupoles ended when I joined STL, to do more traditional plasma physics.

In 1986, I joined the UCLA Plasma Physics Laboratory (UCLA-PPL) for ionospheric and auroral research. There, I was contacted (1994) by Paul Kelly who was developing sensitive ion traps for biophysical and medical applications. P. Kelly was particularly interested in the early history of the Paul trap. He arranged a visit that included D.B. Langmuir, H. Shelton, and myself. Afterwards, I even attend several American Society for Mass Spectrometry (ASMS) meetings, giving poster papers [26,27], while learning all that had happened over the ensuing 30 years! I found that the real breakthrough was the use of electron multipliers, to sense the excited extracted ions with mass resolutions of parts per 10 000. These were the great contributions of John Todd [28], Ray March [19], Peter Dawson [28], and others who brought the Paul trap from a "solution looking for a job" into a highly useful and important piece of scientificanalytical equipment. I still don't know who did what [29]. That is not my task, for as one wag once said; "Physicists may have invented quantum mechanics, but only the chemists use it!" A similar statement can now be applied to the Paul trap!

It is my opinion that the Paul trap is still in its infancy. The trapping of atoms and their forming of ordered damped arrays, sensed with resonant lasers, has already been mentioned. The ac quadrupole field makes a three dimensional potential well which can be reference to an external ground (not possible in a Penning trap). Both positive and negative ions can be simultaneously contained (see Fig. 2, also not possible in Penning traps). Particles and ions can be trapped and resonated at high pressures, an area of important future research.

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## **References**

- [1] Encyclopedia Britannica, Vol. 15, Encyclopedia, Chicago, 1960, p. 335.
- [2] W. Paul, H. Steinwedel, Z Naturfocsch. 8a (1953) 448.
- [3] W. Paul, M. Raether, Zeit. fur Physik 140 (1955) 262.
- [4] W. Paul, H.P. Reinhard, U. von Zahn, Zeit. fur Physik 152 (1958) 143.
- [5] N.W. McLachlan, Theory and Application of Mathieu Functions, Oxford University Press, New York, 1947.
- [6] J.G. Fenn, D.A. Bayne, B.D. Sinclair, Am. J. Phys. 66 (1998) 981.
- [7] H. Shelton, R.F. Wuerker, R.V. Langmuir, Bul. Am. Phys. Soc. II 2 (1957) 375.
- [8] H.C. Corben, Bul. Am. Phys. Soc. II 2 (1957) 375.
- [9] H. Shelton, C.D. Hendricks, R.F. Wuerker, J. Appl. Phys. 31 (1960) 1243.
- [10] J.F. Friichtenicht, Rev. Sci. Instrum. 33 (1962) 209.
- [11] D.G. Becker, J.F. Friichtenicht, B. Hamermesh, R.V. Langmuir, Rev. Sci. Instrum. 36 (1965) 1480.
- [12] R.F. Wuerker, H. Shelton, R.V. Langmuir, J. Appl. Phys. 30 (1959) 342.
- [13] R.F. Wuerker, H.M. Goldenberg, R.V. Langmuir, J. Appl. Phys. 30 (1959) 441.
- [14] H. Straubel, Angewandte P. 18 (1955) 506.
- [15] N.R. Wetten, J. Vac. Soc. Technol. 11 (1974) 515.
- [16] C.D. Hendricks et al. Bul. Am. Phys. Soc., Plasma Division 41 (1996) 1441.
- [17] W. Paul, Rev. Mod. Phys. 62 (1990) 531.
- [18] F. Diedrich, E. Chen, J.W. Quint, H. Walther, Phys. Rev. Lett. 59 (1989) 2931.
- [19] R.E. March, R.J. Hughes, Quadrupole Storage Mass Spectrometry, Wiley, New York, 1989.
- [20] E. Fisher, "Die Driedimensional Stabilisierung von Ladungstrager in einem Vierpolefield," thesis, University of Bonn, Germany, 1958.
- [21] E. Fisher, Z. Phys. 156 (1959) 1.
- [22] R.V. Pound, W.D. Knight, Rev. Sci. Instrum. 21 (1950) 219.
- [23] R. Adler, IEEE Trans. Elect. Devices 59 (1963) 1.
- [24] C.L. Cuccia, Electronics 26 (1953) 130.
- [25] D.B. Langmuir, R.V. Langmuir, H. Shelton, R.F. Wuerker, Containment Device, 1962, U.S. Patent 3,065,640.
- [26] P. Kelly et al., Proceedings at the 42nd ASMS Conference, Chicago, IL, May 29–June 3, 1994, p. 709.
- [27] R.F. Wuerker, Proceedings of the 43rd ASMS Conference, Atlanta, GA, May 21–27, 1994.
- [28] J.F. Todd, G. Lawson, R.F. Bonner, in P.H. Dawson (Ed.), Quadrupole Mass Spectrometry and its Applications, AIP, Woodbury, 1976, Chap. VII.
- [29] P.H. Dawson, P.K. Ghosh, Ion Traps, Oxford Science Publications, Oxford, 1995.